Chapter 9

Rotational Motion II Chapter Review

EQUATIONS:

- $\Gamma = \boldsymbol{r} x \boldsymbol{F}$ [This is the formal definition of torque, where $\boldsymbol{r}$ is a position vector that extends *from* the point about which you are taking the torque *to* the point where the force $\boldsymbol{F}$ acts.]

- $|\Gamma| = |r||F| \sin\theta$ [This defines the magnitude of a torque taken using vectors characterized in polar notation.]

- $|\Gamma| = |r_{\perp}||F|$ [This defines the magnitude of a torque using the force magnitude $\boldsymbol{F}$ and the component of the $\boldsymbol{r}$ vector that is perpendicular to $\boldsymbol{F}$. Often, this is the easiest way to calculate a torque because $r_{\perp}$ turns out to be *the shortest distance between the line of the force* $\boldsymbol{F}$ *and the point about which the torque is being taken*.]

- $|\Gamma| = |r||F_{\perp}|$ [This defines the magnitude of a torque using the magnitude of $\boldsymbol{r}$ and the component of $\boldsymbol{F}$ that is perpendicular to $\boldsymbol{r}$.]

- $\Gamma_{net\ about\ point\ P} = I_P \alpha$ [This is the rotational version of Newton's Second Law. It states that the net torque acting about a point P is equal to the product of the moment of inertia I_P about the point and the angular acceleration α of the body about that point.]

- $KE_{rot\ about\ axis\ P} = \frac{1}{2} I_P \omega^2$ [This is the rotational version of kinetic energy. It states that the kinetic energy associated with a body rotating about axis P is equal to one-half the product of the body's moment of inertia I_P about that point and the square of the body's angular velocity ω about that point.]

- $\boldsymbol{L}_{mass\ A\ about\ pt\ P} = \boldsymbol{r}_P \ x \ \boldsymbol{p}_A$ [This is the rotational counterpart to momentum, or *angular momentum*. In translational parameters, the relationship states that the angular momentum L about a point P is equal to the cross product of a position vector $\boldsymbol{r}$, defined as a line *from* the point about which the angular momentum is being taken *to* the body's position, and the body's translational momentum $m\boldsymbol{v}$. As this is a cross product, you can either determine this quantity using the definition approach, an $r_{\perp}$ approach, or an $F_{\perp}$ approach.]

- $L_{mass\ A\ about\ point\ P} = I_P \omega$ [This is the *magnitude* of the angular momentum calculated using rotational parameters. The direction of the angular momentum vector is the same as the direction of the angular velocity vector, or positive if the rotation is counterclockwise and negative if clockwise.]

- $\Sigma L_1 + \Sigma \Gamma_{net,external}\Delta t = \Sigma L_2$ [This is the rotational counterpart to the modified conservation of momentum equation. It states that the total angular momentum associated with the motion of a system of particles at a given point in time will equal the total angular momentum in the system at some later point in time *if* there are no external rotational impulses $\Gamma_{net,ext}\Delta t$ acting on the system between the two points in time. In practice, this latter condition is rarely met and the expression usually reads $\Sigma L_1 = \Sigma L_2$.]

- $\pm f\Delta t = (mv_2 - mv_1)$ *and/or* $\mp f\Delta t = \dfrac{I\omega_2 - I\omega_1}{R}$ [These equations will come in handy if you ever encounter a problem in which there is FRICTIONAL SLIPPING going on (in such problems, you will normally be asked to determine some *final velocity*). None of the conservation approaches work, so you have to go back to first causes (i.e., Newton's Second Law in its most elementary form). As the common force will always be friction acting over some time interval Δt, the trick is to generate two N.S.L. expressions in impulse form (whether these are both translational, both rotational, or one translational and one rotational expression will depend upon the system). Having grouped the unknowns f and Δt into one unknown ($f\Delta t$) in both expressions, the $f\Delta t$*'s* can be eliminated and the problem solved.]

COMMENTS, HINTS, and THINGS to be aware of:

- One of the nice things about this chapter is that it is essentially a review of all of the approaches and concepts you've run into to date. The problem is that there is a huge amount of material. What you are about to see are some hopefully helpful hints as to how to deal with this.

- If you see any problem that has a **collision**, an **explosion**, or any other situation in which one piece of a **system** changes its motion due to its **interaction** with another piece of the system (assuming all of the pieces within the system can move freely), the first approach you should consider is **conservation of** either **momentum or angular momentum**. Just as small external forces can be ignored *through a collision*, small external torques can be ignored *through a collision* and the total angular momentum of the system before and after the collision will be the same. In cases like this, *angular momentum* is said to be *conserved through the collision.*

- Don't forget, a **point mass** moving in a straight line can have **angular momentum**, relative to a fixed point, even though there appears to be no angular motion happening. That quantity will be $(mv)r_{\perp}$, where $r_{\perp}$ is the shortest distance between the line of motion and the point about which you are calculating the angular momentum quantity.

- Whenever you have a **collision** or **explosion** problem, *never* assume that **mechanical energy** is conserved.

- In a system in which there is both rotation and translation, you must include both rotational and translational kinetic energy in the same **conservation of energy expression**.

- In the modified conservation of energy expression, you will essentially never see an **extraneous work** quantity W_{ext} associated with work done by a **torque**, but you will see similar work quantities associated with forces such as friction.

- If you are **looking for** an **acceleration or angular acceleration**, always think first of Newton's Second Law. Kinematics is also possible, but N.S.L. is usually your first, best bet.

- If you are **looking for** a **velocity or angular velocity**, and if there is motion in which potential energy is changing (i.e., a body changes vertical position in a gravitational field, or a spring goes sproing), always think first about energy considerations. Kinematics is also possible, as is N.S.L. if the body is moving in a circular path, but energy considerations are usually your first, best bet.

- If there is a **collision** or **protracted force interaction** between the pieces of the system and you are asked either for a *force of interaction*, a *velocity*, or an *angular velocity*, always think first of **conservation of momentum** or **conservation of angular momentum**.

- The word acceleration (or velocity) *without* the word ANGULAR in front of it ALWAYS denotes *translational* acceleration (or velocity).

- A point of confusion for a lot of people comes from the fact that there is rotational kinetic energy as well as translational kinetic energy wrapped up in the conservation of energy expression, but there are only momentum quantities in the conservation of momentum expression and only angular momentum quantities in the conservation of angular momentum expression. **DON'T MIX momentum** and **angular momentum quantities** in the same, single, governing expression for a situation. They are two different things (whereas energy is energy whether it is rotational or translational in nature).

- Be careful about **unembedding signs** when using the rotational counterpart of N.S.L. If you think a body's angular acceleration is counterclockwise, the sign of its angular acceleration term should be positive. If clockwise, it's negative.

- Don't forget, when a **rope** is strung **over** a **massive pulley**, the tension on either side of the pulley will be different.

- Remember, you can either look at the motion of a **body** that is **both rotating and translating** as a *rotation about and translation of* the body's center of mass, or as an instantaneous, *pure rotation* about the only point on the mass that is not instantaneously moving (i.e., the point of contact with the support over which the mass is rolling). This is useful both in N.S.L. problems and in conservation of energy problems. Either approach will work in the analysis of a problem (though one is often easier than the other, depending), but don't mix 'em.

- **Don't be buffaloed** by problems that look familiar but are different. A favorite AP trick is to give you a problem you've seen before, but turned around. You'd be surprised how many people get confused when the incline is presented sloping from the right to the left, instead of from the left to the right as is usually shown in texts. Remember, what you have at your disposal is a series of approaches that will provide you with mathematical relationships that are true (i.e., one side of the expression will equal the other side of the expression). You know how to use those approaches. Use them from scratch. Don't get hung up trying to make a test problem fit the form of a problem you've seen before.